1-7 Absolute Value Equations and Inequalities

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I CAN... write and solve absolute value equations and inequalities.

Activity Assess

MODEL & DISCUSS

Amelia is participating in a 60-mile spin-a-thon. Her spin bike keeps track of the simulated number of miles she travels. She plans to take a 15-minute break within 5 miles of riding 30 miles.

Spin-a-thon Schedule	
Event	Time
Start spinning	10:00 A.M.
Stop for break	■
Resume spinning	■

A. Write a compound inequality that models the number of miles Amelia spins before taking a break.

B. How is the number of miles Amelia spins before she takes a break related to the amount of time before she takes a break?

C. **Make Sense and Persevere** About how many hours will Amelia spin before she takes a break? Discuss how you could use your mathematical model to complete the spin-a-thon schedule.

ESSENTIAL QUESTION Why does the solution for an absolute value equation or inequality typically result in a pair of equations or inequalities?

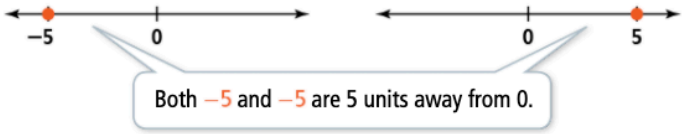
EXAMPLE 1 Understand Absolute Value Equations

A. What is the value of x in $7 = |x| + 2$?

Solve for x by isolating the absolute value expression on one side of the equation.

$$7 = |x| + 2$$
$$7 - 2 = |x| + 2 - 2$$
$$5 = |x|$$

Use the Subtraction Property of Equality.



Both −5 and −5 are 5 units away from 0.

The solutions are $x = -5$ and $x = 5$.

Check the solutions.

$$7 \stackrel{?}{=} |-5| + 2 \qquad 7 \stackrel{?}{=} |5| + 2$$
$$\stackrel{?}{=} 5 + 2 \qquad \stackrel{?}{=} 5 + 2$$
$$= 7 \checkmark \qquad = 7 \checkmark$$

STUDY TIP
The absolute value of a number is its distance from 0 on a number line.

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USE STRUCTURE
When solving an absolute value equation in the form $|ax + b| = c$, use two different equations to find the solutions, $ax + b = c$ and $ax + b = -c$.

EXAMPLE 1 CONTINUED

B. What is the value of x in $|2x - 3| = 1$?

Write and solve equations for the two possibilities:

$2x - 3$ is positive.

$$2x - 3 = 1$$
$$2x - 3 + 3 = 1 + 3$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

$2x - 3$ is negative.

$$2x - 3 = -1$$
$$2x - 3 + 3 = -1 + 3$$
$$2x = 2$$
$$\frac{2x}{2} = \frac{2}{2}$$
$$x = 1$$

The expression inside the absolute value symbol can be positive or negative. So the expression $2x - 3$ can be equal to 1 or -1.

The solutions are $x = 2$ and $x = 1$.

C. What is the value of x in $3|x + 6| + 8 = 5$?

Step 1 Isolate the absolute value expression.

$$3|x + 6| + 8 - 8 = 5 - 8$$
$$3|x + 6| = -3$$
$$\frac{3|x + 6|}{3} = \frac{-3}{3}$$

Step 2 Solve for x.

$$|x + 6| = -1$$

This equation has no solution.

The absolute value of a number is a distance and cannot be negative.

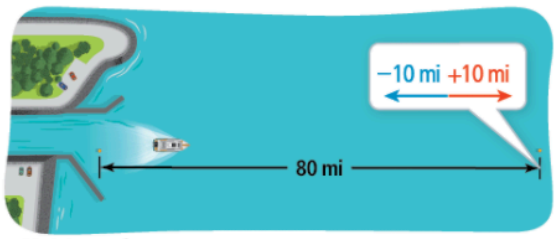
Try It! **1.** Solve each equation.

a. $6 = |x| - 2$ **b.** $2|x + 5| = 4$ **c.** $|3x - 6| = 12$

EXAMPLE 2 Apply an Absolute Value Equation

STUDY TIP
You can use an absolute value equation to model a quantity "plus or minus" another quantity.

The cruising speed of Kennedy's boat is 25 mi/h. She plans to cruise at this speed for the distances shown in the diagram.



Not to scale

A. What equation models the number of hours x that Kennedy will travel?

The distance Kennedy actually travels

$$\underbrace{|25x - 80|}_{\text{Final distance from the 80-mile point}} = 10$$

10 miles from the 80-mile point

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EXAMPLE 2 CONTINUED

B. What are the minimum number and maximum number of hours Kennedy will travel?

Write and solve equations for the two possibilities.

If Kennedy travels plus 10 miles, the absolute value expression is positive.

$$25x - 80 = 10$$
$$25x - 80 + 80 = 10 + 80$$
$$25x = 90$$
$$\frac{25x}{25} = \frac{90}{25}$$
$$x = 3.6$$

If Kennedy travels minus 10 miles, the absolute value expression is negative.

$$25x - 80 = -10$$
$$25x - 80 + 80 = -10 + 80$$
$$25x = 70$$
$$\frac{25x}{25} = \frac{70}{25}$$
$$x = 2.8$$

The solutions are $x = 3.6$ and $x = 2.8$.

Kennedy will travel at least 2.8 hours and at most 3.6 hours.

Try It! **2.** What will be the minimum and maximum time that Kennedy will travel if she resets her cruising speed to 20 mi/h?

CONCEPTUAL UNDERSTANDING

EXAMPLE 3 Understand Absolute Value Inequalities

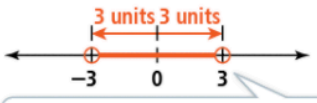
What are the solutions of an absolute value inequality?

COMMON ERROR

Remember to look at the > and < symbols when solving absolute value inequalities. Inequalities with absolute value have to be set up differently to solve if it is an "and" situation vs. an "or" situation.

Solve and graph two absolute value inequalities.

A.

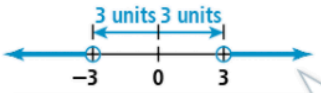


The distance between x and 0 must be less than 3, so the values 3 units to the right and 3 units to the left are solutions.

$|x| < 3$

$|x| < 3$ is equivalent to the compound inequality $x < 3$ *and* $x > -3$, which can also be written as $-3 < x < 3$.

B.



The distance between x and 0 must be greater than 3. So positive values of x must be greater than 3, and negative values of x must be less than -3.

$|x| > 3$

$|x| > 3$ is equivalent to the compound inequality $x < -3$ *or* $x > 3$.

Try It! **3.** Solve and graph the solutions of each inequality.

a. $|x| > 15$ **b.** $|x| \leq 7$

APPLICATION

EXAMPLE 4 Write an Absolute Value Inequality

Members of the debate team are traveling to a tournament, where they will stay in a hotel for 4 nights. The total cost for each member must be within $20 of $175. Which of the hotels shown can they consider?

Hotel Room Costs*

*Per person per night

Formulate Write an absolute value inequality to represent the situation.

Let x be the cost per night of a hotel room.

The difference between total cost and $175 is less than or equal to $20.

$|4x - 175| \leq 20$

Compute Solve the inequality to find the maximum and minimum hotel cost for each team member.

Maximum Cost	Minimum Cost
$4x - 175 \leq 20$	$4x - 175 \geq -20$
$4x - 175 + 175 \leq 20 + 175$	$4x - 175 + 175 \geq -20 + 175$
$4x \leq 195$	$4x \geq 155$
$\frac{4x}{4} \leq \frac{195}{4}$	$\frac{4x}{4} \geq \frac{155}{4}$
$x \leq 48.75$	$x \geq 38.75$

Interpret The cost of the hotel room can be between $38.75 and $48.75, inclusive.

The debate team can consider Hotel A or Hotel B.

Try It! **4.** If the debate team increased their limit to $200 plus or minus $20, would they be able to afford Hotel D at $55 per night? Explain.

CONCEPT SUMMARY Absolute Value Equations and Inequalities

WORDS

Absolute Value Equations

To solve an absolute value equation, isolate the absolute value expression. Then write two equations and solve.

Absolute Value Inequalities

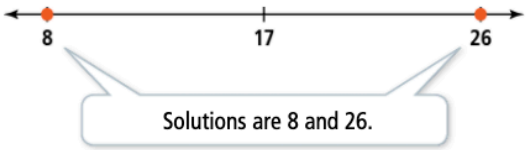
If an inequality uses $<$ or $\leq$ and these symbols point to the variable in the solution, the solution uses "and". If an inequality uses $>$ or $\geq$ and these symbols point to the variable in the solution, the solution uses "or".

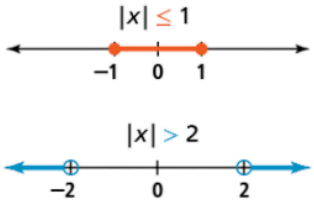
ALGEBRA

$$2|x - 17| = 18$$
$$|x - 17| = 9$$
$$x - 17 = -9 \qquad x - 17 = 9$$
$$x = 8 \qquad x = 26$$

$$|x| \leq 1$$
$$x \geq -1 \text{ and } x \leq 1$$
$$|x| > 2$$
$$x < -2 \text{ or } x > 2$$

DIAGRAMS





Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** Why does the solution for an absolute value equation or inequality typically result in a pair of equations or inequalities?

2. **Reason** How is solving an absolute value equation similar to solving an equation that does not involve absolute value? How is it different?

3. **Vocabulary** Describe how you would explain to another student why the *absolute value* of a number cannot be negative.

4. **Error Analysis** Yumiko solved $|x| > 5$ by solving $x > -5$ and $x < 5$. Explain the error Yumiko made.

Do You KNOW HOW?

Solve each absolute value equation.

5. $5 = |x| + 3$

6. $|2x - 8| = 16$

Solve each absolute value inequality. Graph the solution.

7. $|3x - 6| \geq 9$

8. $|4x - 12| \leq 20$

9. On a road trip, Andrew plans to use his cruise control for 125 mi, plus or minus 20 mi. Write and solve an equation to find the minimum and maximum number of hours for Andrew's road trip.

PRACTICE & PROBLEM SOLVING

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Practice Tutorial

Additional Exercises Available Online

UNDERSTAND

10. Make Sense and Persevere Sasha is solving the absolute value equation $|2x| + 4 = 8$. What is the first step she should take?

11. Use Structure The absolute value inequality $5 \le |x| - n$ is graphed below. What is the value of n?

12. Error Analysis Describe and correct the error a student made when solving $2|x| < 16$.

Solve $2|x| < 16$.

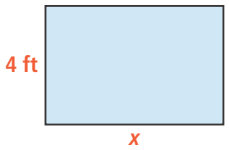
$2|x| < 16$

$\frac{2|x|}{2} < \frac{16}{2}$ Divide both sides by 2.

$|x| < 8$ Simplify.

$x < 8$ or $x > -8$ Rewrite using "or."

13. Mathematical Connections Jack wants to model a situation where the perimeter of the rectangle below is 6 ft plus or minus 1.5 ft.



Because he is modeling a length "plus or minus" another length, he decides to use an absolute value equation for his model. Do you agree with his decision? Explain your reasoning.

14. Higher Order Thinking Let a, b, c, and x be real numbers.

a. How is solving $|ax| + b = c$ different from solving $|ax + b| = c$?

b. How is solving $|ax| + b \le c$ different from solving $|ax + b| \ge c$?

PRACTICE

Solve each absolute value equation. SEE EXAMPLE 1

15. $2 = |x| - 1$

16. $|x| - 4 = 9$

17. $14 = |x| + 2$

18. $|x| + 4 = -9$

19. $|-2x + 8| = 20$

20. $|x - 4| = 9$

21. $2|x + 8| = 20$

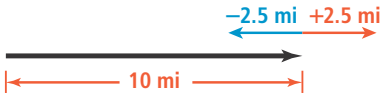
22. $2|x - 8| = 20$

23. $5|x + 3| + 8 = 6$

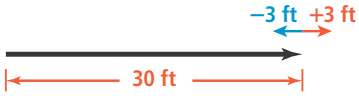
24. $3|x - 2| - 8 = 7$

Write and solve an absolute value equation for the minimum and maximum times for an object moving at the given speed to travel the given distance. (Figures are not to scale.) SEE EXAMPLE 2

25. 5 mi/h



26. 10 ft/s



Solve each absolute value inequality. Graph the solution. SEE EXAMPLES 3 AND 4

27. $2 \le |x| - 8$

28. $-2 > |x| - 8$

29. $|x| + 5 \ge 10$

30. $|x| + 2.4 < 3.6$

31. $|2x + 5| \ge 9$

32. $|2x - 5| < 9$

33. $-2|x + 4| \le -6$

34. $-2|2x + 4| + 10 > -6$

Match each absolute value inequality to the graph that represents its solution. Explain your reasoning. SEE EXAMPLES 3 AND 4

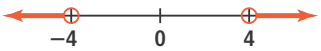
35. $3|x| - 2 \le 10$

36. $2|x| - 1 < 7$

37. $3|2x| + 1 > 25$

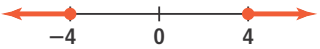
38. $2|4x| - 7 \ge 25$

A.



B.

C.



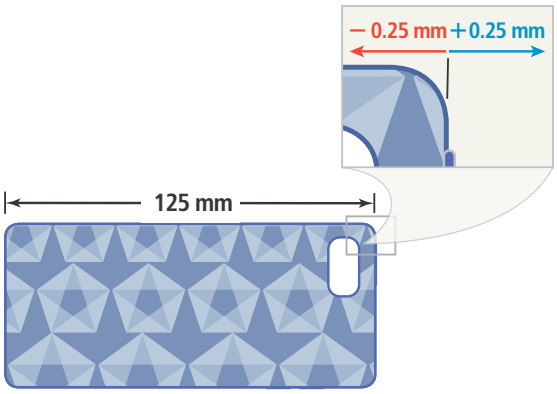
D.

PRACTICE & PROBLEM SOLVING

Practice Tutorial
Mixed Review Available Online

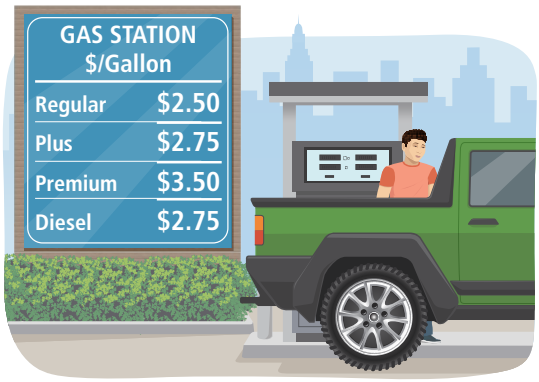
APPLY

39. Make Sense and Persevere A company manufactures cell phone cases. The length of a certain case must be within 0.25 mm of 125 mm, as shown (figure is not to scale). All cases with lengths outside of this range are removed from the inventory. How could you use an absolute value inequality to represent the lengths of all the cases that should be removed? Explain.



40. Construct Arguments Ashton is hosting a banquet. He plans to spend \$400, plus or minus \$50, at a cost of \$25 per guest. Solve $|25x - 400| = 50$ to find the maximum and minimum number of guests. If there can be up to 7 guests at each table, what is the minimum number of tables Ashton should reserve so that every guest has a seat?

41. Model With Mathematics Hugo is pumping regular gas into his truck. Write and solve an absolute value equation to represent how many gallons of gas will be pumped when the total is \$25 plus or minus \$0.50.



ASSESSMENT PRACTICE

42. Arrange steps in the solution to $2|x - 3| + 4 < 12$ in the correct order.

A. $-1 < x < 7$
B. $2|x - 3| + 4 < 12$
C. $2|x - 3| < 8$
D. $-4 < x - 3 < 4$
E. $|x - 3| < 4$

43. SAT/ACT What is the solution of $|4x - 6| = 2$?

Ⓐ $x = 1, x = 2$
Ⓑ $x = -1, x = 2$
Ⓒ $x = 1, x = -2$
Ⓓ $x = -1, x = -2$
Ⓔ $x = -2, x = 2$

44. Performance Task A road sign shows a vehicle's speed as the vehicle passes.

Part A The sign blinks for vehicles traveling within 5 mi/h of the speed limit. Write and solve an absolute value inequality to find the minimum and maximum speeds of an oncoming vehicle that will cause the sign to blink.

Part B Another sign blinks when it detects a vehicle traveling within 2 mi/h of a 35 mi/h speed limit. Write and solve an absolute value inequality to represent the speeds of the vehicles that cause the sign to blink.

Part C The sign is programmed to blink using absolute value inequalities of the form $|x - a| \le b$ and $|x - a| \ge b$. Which of these formulas is used to program the sign for cars traveling either 5 mi/h above or below the 20 mi/h speed limit? What are the values of a and b? Explain.

11. $n = 2$

13. No, an absolute value equation will not work because the value of x would have to be negative for the perimeter to be 6 feet plus or minus 1.5 feet, and an absolute value expression cannot have a negative value.

15. $x = -3$, $x = 3$

17. $x = -12$, $x = 12$

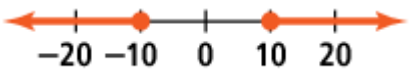
19. $x = -6$, $x = 14$

21. $x = -18$, $x = 2$

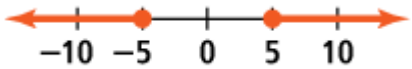
23. no solution

25. $|5x - 10| = 2.5$; minimum: 1.5 hours; maximum: 2.5 hours

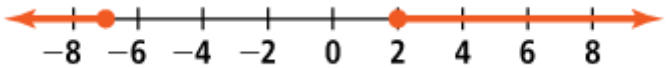
27. $x \leq -10$ or $x \geq 10$;



29. $x \leq -5$ or $x \geq 5$;



31. $x \leq -7$ or $x \geq 2$;



33. $x \leq -7$ or $x \geq -1$;

35. B

37. A

39. The difference between the actual length of a case x and 125 mm must, at most, be within 0.25 mm. The inequality $|x - 125| \leq 0.25$ can be used to represent this acceptable range of lengths. Therefore, the range of lengths of cases that should be removed can be represented by the absolute value inequality $|x - 125| > 0.25$.

41. $|2.50x - 25| \leq 0.50$; $9.8 \leq x \leq 10.2$; Between 9.8 and 10.2 gallons will be pumped.

43. A